Control System Design in Mechatronic Systems

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Control of Mechatronic Systems
Three-Part Web Cast

- Part 1
  - Control System Introduction
- Part 2
  - Control System Stability and Performance
- Part 3
  - Control System Design
Web Cast Introduction

Other Components
Communications

Operator Interface
Human Factors

Computation
Software, Electronics

Plant
Design

Modern Mechatronic System

Actuation
Power Modulation
Energy Conversion

Instrumentation
Energy Conversion
Signal Processing

Physical System
Mechanical, Fluid,
Thermal, Chemical,
Electrical, Mixed

Plant Dynamics
& Control Structure

Plant Dynamics
& Control Structure

Control of Mechatronic Systems
Challenge to Engineers

• Control Design and Implementation is still the domain of the specialist.
• Controls and Electronics are still viewed as afterthought add-ons.
• Very few practicing engineers perform any kind of physical and mathematical modeling.
• Mathematics is a subject that is not viewed as enhancing one’s engineering skills but as an obstacle to avoid.
• Very few engineers have the balance between analysis and hardware essential for success in Mechatronics.
Part 1: Control System Introduction

- Process or Plant
- Process Inputs
  - Manipulated Inputs
  - Disturbance Inputs
- Response Variables

Control systems are an integral part of the overall system and not after-thought add-ons! The earlier the issues of control are introduced into the design process, the better!

Why Controls?
- Command Following
- Disturbance Rejection
- Sensitivity Reduction

Everything Needs Controls for Optimum Functioning!
• Classification of Control System Types
  – **Open-Loop**
    • Basic
    • Input-Compensated Feedforward
      – Disturbance-Compensated
      – Command-Compensated
  – **Closed-Loop (Feedback)**
    • Classical
      – Root-Locus
      – Frequency Response
    • Modern (State-Space)
    • Advanced
      – e.g., Adaptive, Nonlinear, Fuzzy Logic
Basic Open-Loop Control System

Satisfactory if:
- disturbances are not too great
- changes in the desire value are not too severe
- performance specifications are not too stringent
**Open-Loop Input-Compensated Feedforward Control: Disturbance-Compensated**

- Measure the disturbance
- Estimate the effect of the disturbance on the controlled variable and compensate for it
Open-Loop Input-Compensated Feedforward Control: Command-Compensated

Based on the knowledge of plant characteristics, the desired value input is augmented by the command compensator to produce improved performance.
**Closed-Loop (Feedback) Control System**

Flow of Energy and/or Material

- Control Effector
- Plant
- Controlled Variable Sensor
- Control Director
- Desired Value of Controlled Variable

**Open-Loop Control System is converted to a Closed-Loop Control System by adding:**

- measurement of the controlled variable
- comparison of the measured and desired values of the controlled variable
Open-Loop Control System

Disturbance Input Element

Desired Value

Reference Input Element

Controller

Manipulated Variable

Plant

Controlled Variable

Closed-Loop Control System

Actuating Signal

Feedback Signal

Sensor Error

Sensor (Feedback Element)

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• **Basic Benefits of Feedback Control**
  
  – Cause the controlled variable to accurately follow the desired variable; corrective action occurs as soon as the controlled variable deviates from the command.

  – Greatly reduces the effect on the controlled variable of all external disturbances in the forward path. It is ineffective in reducing the effect of disturbances in the feedback path (e.g., those associated with the sensor), and disturbances outside the loop (e.g., those associated with the reference input element).

  – Is tolerant of variations (due to wear, aging, environmental effects, etc.) in hardware parameters of components in the forward path, but not those in the feedback path (e.g., sensor) or outside the loop (e.g., reference input element).
– Can give a closed-loop response speed much greater than that of the components from which they are constructed.

• **Inherent Disadvantages of Feedback Control**
  – No corrective action is taken until after a deviation in the controlled variable occurs. Thus, perfect control, where the controlled variable does not deviate from the set point during load or set-point changes, is theoretically impossible.
  – It does not provide predictive control action to compensate for the effects of known or measurable disturbances.
– It may not be satisfactory for processes with large time constants and/or long time delays. If large and frequent disturbances occur, the process may operate continually in a transient state and never attain the desired steady state.

– In some applications, the controlled variable cannot be measured on line and, consequently, feedback control is not feasible.

• For situations in which feedback control by itself is not satisfactory, significant improvements in control can be achieved by adding feedforward control.
Stability

• All feedback systems can become unstable if improperly designed.
• In all real-world components there is some kind of lagging behavior between the input and output, characterized by $\tau$’s and $\omega_n$’s.
• Instantaneous response is impossible in the real world!
• Instability in a feedback control system results from an improper balance between the strength of the corrective action and the system dynamic lags.
• If a system in equilibrium is momentarily excited by command and/or disturbance inputs and those inputs are then removed, the system must return to equilibrium if it is to be called **absolutely stable.**

• If action persists indefinitely after excitation is removed, the system is judged **absolutely unstable.**

• If a system is stable, how close is it to becoming unstable? **Relative stability indicators are gain margin and phase margin.**

• If we want to make valid stability predictions, we must include enough dynamics in the system model so that the closed-loop system differential equation is at least third order.
  
  – An exception to this rule involves systems with dead times, where instability can occur when the dynamics (other than the dead time itself) are zero, first, or second order.
The analytical study of stability becomes a study of the stability of the solutions of the closed-loop system’s differential equations.

A complete and general stability theory is based on the locations in the complex plane of the roots of the closed-loop system characteristic equation, stable systems having all of their roots in the LHP.
Root Locus method gives information about closed-loop behavior given the open-loop transfer function.

The root locus is a plot of the poles of the closed-loop transfer function as any single parameter varies from 0 to $\infty$.

$$\frac{B}{E}(s) = \frac{K}{s(\tau_1 s + 1)(\tau_2 s + 1)}$$
Feedback Control System Block Diagram

Plausibility Demonstration for the Nyquist Stability Criterion
\[ \frac{B}{E}(i\omega) = G_1 G_2 H(i\omega) \]

Polar Plot of Open-Loop Frequency Response

Simplified Version of Nyquist Stability Criterion
The diagram illustrates the gain margin (1/GM) and phase margin (PM) of a control system. The gain margin is defined as the reciprocal of the gain at the crossover frequency, while the phase margin is the difference between the phase angle at the crossover frequency and -180°.
\[
G(s) = \frac{85(s + 1)(s^2 + 2s + 43.25)}{s^2(s^2 + 2s + 82)(s^2 + 2s + 101)} \\
= \frac{85(s + 1)(s + 1\pm 6.5j)}{s^2(s + 1\pm 9j)(s + 1\pm 10j)}
\]

Nyquist Plot

Bode Plots
Performance

- Standard performance criteria may be classified as falling into two categories:
  - **Time-Domain Specifications**: Response to steps, ramps, and the like
  - **Frequency-Domain Specifications**: Concerned with certain characteristics of the system frequency response
- Both time-domain and frequency-domain design criteria generally are intended to specify one or the other of:
  - speed of response
  - relative stability
  - steady-state errors
- All performance specifications are meaningless unless the system is absolutely stable.
Closed-Loop Step Response

\[ O_p \triangleq \frac{O}{V} \times 100 \]

- \( T_p \)
- \( T_{r.5\%} \)
- \( T_r \)
- \( \pm 0.05 \text{ V} \)
Effect of Command Severity on Steady-State Error
Typical Closed-Loop Frequency-Response Curves

As noise is generally in a band of frequencies above the dominant frequency band of the true signal, feedback control systems are designed to have a definite passband in order to reproduce the true signal and attenuate noise.

\[
\frac{C}{V}(i\omega) = \frac{AG_1G_2(i\omega)}{1 + G_1G_2H(i\omega)}
\]
Open-Loop Performance Criteria: Gain Margin and Phase Margin

A system must have adequate stability margins. Both a good gain margin and a good phase margin are needed. Useful lower bounds:

\[ \text{GM} > 2.5 \quad \text{PM} > 30^\circ \]
Bode Plot View of Gain Margin and Phase Margin
Desired Shape for Open-Loop Transfer Function

Smooth transition from the low to high-frequency range, i.e., -20 dB/decade slope near the gain crossover frequency

- Gain above this level at low frequencies
- Gain below this level at high frequencies

Frequencies for good command following, disturbance reduction, sensitivity reduction

Sensor noise, unmodeled high-frequency dynamics are significant here.
Part 3: Control System Design

Magnetic Levitation System

Electromagnetic Valve Actuator
For a Camless Automotive Engine
Magnetic Levitation System

Electromagnet
Infrared LED
Levitated Ball

Magnetic Levitation Control Design with LabVIEW

\[ f(x, i) = c \left( \frac{i^2}{x^2} \right) \]

Electromagnet
Ball (mass m)

\[ V_{\text{desired}} \]
\[ V_{\text{bias}} \]
\[ G_c(s) \]
Controller
Current Amplifier
\[ G(s) \]
Magnet + Ball

\[ V_{\text{actual}} \]

\[ H(s) \]
Sensor

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**Electromagnet Actuator**

- Current flowing through the coil windings of the electromagnet generates a magnetic field.
- The ferromagnetic core of the electromagnet provides a low-reluctance path in which the magnetic field is concentrated.
- The magnetic field induces an attractive force on the ferromagnetic ball.

Electromagnetic Force

Proportional to the square of the current and inversely proportional to the square of the gap distance.

\[ f(x, i) = C \left( \frac{i^2}{x^2} \right) \]

Electromagnet \hspace{2cm} Ball (mass m)

\[ i \quad \hat{i} \]

\[ mg \]
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+5V

1 KΩ

62 Ω

Infrared LED

Emitter

+15V

200 KΩ

Detector

Phototransistor

Unity Gain Buffer Op-Amp

e_{sensor}
Magnetic Levitation System Block Diagram

Linear Feedback Control System to Levitate Steel Ball about an Equilibrium Position Corresponding to Equilibrium Gap $x_0$ and Equilibrium Current $i_0$

From Equilibrium:
As $i \uparrow$, $x \downarrow$, & $V_{\text{sensor}} \downarrow$
As $i \downarrow$, $x \uparrow$, & $V_{\text{sensor}} \uparrow$

$$f(x,i) = C \left( \frac{i^2}{x^2} \right)$$

Electromagnet

Ball (mass $m$)

mg
Voltage-to-Current Converter

Assume Ideal Op-Amp Behavior

\[ e^+ = e^- \]

\[ i_M = \left( \frac{R_2}{R_1 + R_2} \right) \left( \frac{1}{R_S} \right) e_{in} \]

\[ R_1 = 49\, \text{K}\Omega, \quad R_2 = 1\, \text{K}\Omega, \quad R_3 = 0.1\, \text{\Omega} \]
Non-Ideal Op-Amp Behavior

\[ e_o = \frac{A}{\tau s + 1} (e^+ - e^-) \]

\[ e_{out} - e_1 = (L_M s + R_M) i \]

\[ e_1 = R_s i \]

\[ e_{out} - e_1 = (L_M s + R_M) \frac{e_1}{R_s} \]

\[ e_{out} = \left( \frac{L_M s + R_M + R_s}{R_s} \right) e_1 \]
Magnetic Levitation System
Control System Design

Equation of Motion:
\[ f(x,i) = C \left( \frac{i^2}{x^2} \right) \]

At Equilibrium:
\[ mg = C \left( \frac{i^2}{X^2} \right) \]

Linearization:
\[
\begin{align*}
C \left( \frac{i^2}{X^2} \right) & \approx C \left( \frac{i^2}{X^2} \right) - C \left( \frac{2i^2}{X^3} \right) \hat{x} + C \left( \frac{2i}{X^2} \right) \hat{\imath} \\
mg & - C \left( \frac{i^2}{X^2} \right) + C \left( \frac{2i^2}{X^3} \right) \hat{x} - C \left( \frac{2i}{X^2} \right) \hat{\imath} \\
mX & = C \left( \frac{2i^2}{X^3} \right) \hat{x} - C \left( \frac{2i}{X^2} \right) \hat{\imath}
\end{align*}
\]
Use of Experimental Testing in Multivariable Linearization

\[ f_m = f(i, y) \]

\[ f_m \approx f(i_0, y_0) + \frac{\partial f}{\partial y}_{i_0, y_0} (y - y_0) + \frac{\partial f}{\partial i}_{i_0, y_0} (i - i_0) \]
\[
\begin{align*}
\mathbf{V}_{\text{desired}} & \xrightarrow{+} \mathbf{G}_c(s) \xrightarrow{+} \mathbf{V}_{\text{bias}} \\
& \xrightarrow{\Sigma} \mathbf{G}_c(s) \mathbf{C} \xrightarrow{\Sigma} \mathbf{Current \ Amplifier} \\
& \xrightarrow{\mathbf{i}} \mathbf{G}(s) \mathbf{Magnet \ + \ Ball} \\
& \mathbf{V}_{\text{actual}} \xrightarrow{\mathbf{H}(s) \mathbf{Sensor}} \mathbf{X}
\end{align*}
\]

\[
\begin{align*}
m &= 0.008 \\
g &= 9.81 \\
x &= 0.003 \\
i &= 0.222
\end{align*}
\]

\[
\begin{align*}
\mathbf{m} \mathbf{g} &= \mathbf{C} \left( \frac{i^2}{x^2} \right) \\
& \Rightarrow \mathbf{C} = 1.4332 \times 10^{-5}
\end{align*}
\]

\[
\begin{align*}
\mathbf{m}x'' &= \mathbf{C} \left( \frac{2i^2}{x^3} \right) \hat{x} - \mathbf{C} \left( \frac{2i}{x^2} \right) \hat{i} \\
& \Rightarrow \frac{\hat{x}}{\hat{i}} = \frac{-88}{s^2 - 6540}
\end{align*}
\]

\[
\mathbf{X} = 6540 \hat{x} - 88 \hat{i}
\]
Open-Loop Transfer Function

\[
\frac{88}{(s^2 - 6540)}(0.2)(4000) = \frac{70400}{s^2 - 6540}
\]

Controller

**PD Controller**

\[
\frac{K_P + K_D s}{\tau s + 1} + \frac{K_D}{\tau} \frac{s + 1}{s + \frac{1}{\tau}}
\]

\[
\tau = 0.002
\]

\[
K_P = 0.3
\]

\[
K_D = 0.003
\]
Survey Questions

• How many of the systems you design have a control system?
• As a mechatronics engineer do you design and implement control systems or are they done by a specialist?
• Is the control system integrated into the mechatronic system design at the beginning of the design process or is it an after-thought add-on?
THANK YOU!