Time Domain & Frequency Domain

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Time Domain & Frequency Domain
Time domain and frequency domain are two ways of looking at the same dynamic system. They are interchangeable, i.e., no information is lost in changing from one domain to another.

They are complementary points of view that lead to a complete, clear understanding of the behavior of a dynamic engineering system.

Roughly speaking, in the time domain we measure how long something takes, whereas in the frequency domain we measure how fast or slow it is.

These are two ways of viewing the same thing!
– When you hear music and see color, you are experiencing the frequency domain. It is all around you, just like the time domain.

– The frequency domain is a kind of hidden companion to our everyday world of time. We describe what happens in the time domain as **temporal** and in the frequency domain as **spectral**.

– Most signals and processes involve both fast and slow components happening at the same time. Frequency domain analysis separates these components and helps to keep track of them.
• **Mechanical Spectrum**
  - Second hand of a clock: 1 rpm
  - Audio CDs: 200 to 500 rpm
  - Dentist’s drill: 400,000 rpm
  - Two-foot diameter tire on a car traveling at 60 mph: 840 rpm
  - Earth’s rotation: 0.00069 rpm (1000 mph at the equator!)
  - $1 \text{ Hz} = 60 \text{ rpm} = 2\pi \text{ rad/sec} = 6.28 \text{ rad/sec}$. All have dimensions $1/\text{time}$. 
Mechanical Spectrum

Electromagnetic Spectrum
• **Electromagnetic Spectrum**
  – Electromagnetic effects can be described in the frequency domain as well.
  – Electromagnetic waves travel at the speed of light \( c \), which depends on the medium (fastest in a vacuum, slower in other media).
  – The frequency of vibration \( f \) depends on the wavelength \( \lambda \) of the electromagnetic phenomenon and the speed of propagation \( c \) of the medium according to \( f = c/\lambda \).
  – Long-wavelength electromagnetic waves are radio waves (see spectrum diagram). Frequencies range from a few kHz to 300 GHz.
  – Higher frequencies are emitted by thermal motion, which we call infrared radiation.
- Frequencies of visible light range from 440 THz (red light) to 730 THz (violet light).
- Humans perceive different frequencies within the visible light spectrum as different colors. Unlike the ear, the eye has a nonlinear response to combinations of frequencies.
Electromagnetic Spectrum

- Radio
- Microwave
- Infrared
- Visible
- Ultraviolet
- X-Ray
- Gamma Ray

Long wave length → Short wave length
Low frequency → High frequency
Low energy → High energy
Radio Spectrum

Time Domain & Frequency Domain
• **Time Domain**
  
  – The time domain is a record of the response of a dynamic system, as indicated by some measured parameter, as a function of time. This is the traditional way of observing the output of a dynamic system.

  – An example of time response is the displacement of the mass of the spring-mass-damper system versus time in response to the sudden placement of an additional mass (here 50% of the attached mass) on the attached mass. The resulting response is the step response of the system due to the sudden application of a constant force to the attached mass equal to the weight of the additional mass. Typically when we investigate the performance of a dynamic system we use as the input to the system a step input.
• **Frequency Domain**
  
  – Over one hundred years ago, Jean Baptiste Fourier showed that any waveform that exists in the real world can be generated by adding up sine waves.
  
  – By picking the amplitudes, frequencies, and phases of these sine waves, one can generate a waveform identical to the desired signal.
  
  – While the situation presented on the next page is contrived, it does illustrate the idea. On the left is a “real-world” signal and on the right are three signals, the sum of which is the same as the “real-world” signal.
A more convincing example is to observe that a square wave can be represented by a series of sine waves of different amplitudes, frequencies, and phase angles. In the diagram below, a square wave has been approximated with only two sine waves. As more sine waves are added to the series, the approximation becomes better and better.
– Any real-world signal can be broken down into a sum of sine waves and this combination of sine waves is unique. Any real-world signal can be represented by only one combination of sine waves.

– In the diagram, a waveform is represented as the sum of two sine waves.
– In figure (a) is a three-dimensional graph of this addition of sine waves. The three axes are time, amplitude, and frequency. The time and amplitude axes are familiar from the time domain. The third axis, frequency, allows us to visually separate the sine waves that add to give us the complex waveform.

– If we view this three-dimensional graph along the frequency axis, we get the view shown in figure (b). This is the time-domain view of the sine waves. Adding them together at each instant of time gives the original waveform.
– Now view the three-dimensional graph along the time axis, as in figure (c). Here we have axes of amplitude versus frequency. This is what is called the frequency domain. Every sine wave we separated from the input appears as a vertical line. Its height represents its amplitude and its position represents its frequency. We know each line represents a sine wave and so we have uniquely characterized our input signal in the frequency domain. This frequency domain representation of our signal is called the spectrum of the signal. Each sine wave line of the spectrum is called a component of the total signal.
– It is most important to understand that we have neither gained nor lost information, we are just representing it differently.

– You can now see why a sine wave is the second important signal, the step input being the other, used to excite a dynamic system.

– Since any real-world signal can be represented by the sum of sine waves, if we can predict the response of a system to a sine wave input of varying frequency, amplitude, and phase angle, then we can predict the response of the system to any real-world signal once we know the frequency spectrum of that real-world signal.
LabVIEW
Time Domain
Frequency Domain
Exercise
LabVIEW Block Diagram